

Sheet 6

[1] Consider the unit-step response of the unity-feedback control system whose open-loop transfer function is $G(s) = \frac{1}{s(s+1)}$

obtain

rise time, peak time, maximum overshoot and settling time.

$$\frac{C(s)}{R(s)} = \frac{\overset{\text{sol}}{1}}{s(s+1)} \div 1 + \frac{1}{s(s+1)} = \frac{1}{s^2 + s + 1}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n = 1, \quad \zeta = 0.5$$

$$t_r = \frac{\pi - \cos^{-1} \zeta}{\omega_n \sqrt{1 - \zeta^2}} \quad \text{or} \quad M_p = e^{-\frac{2\pi\zeta}{\sqrt{1 - \zeta^2}}} = 0.256 \text{ sec}$$

$$t_s = \frac{4}{\zeta\omega_n} = 8 \text{ sec} \quad \left| \quad t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = 3.6 \text{ sec} \right.$$

[2] Consider the closed-loop system given by:-

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

→ Determine the values of ζ and ω_n so that the system responds to a step input with approximately 5% overshoot and with settling of 2 sec (use the 2% criterion)

$$M_p = 5\% \quad , \quad t_s = 2 \text{ sec}$$

[Sol]

$$\frac{5}{100} = e^{-\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\zeta = \checkmark$$

$$t_s = \frac{4}{\zeta\omega_n}$$

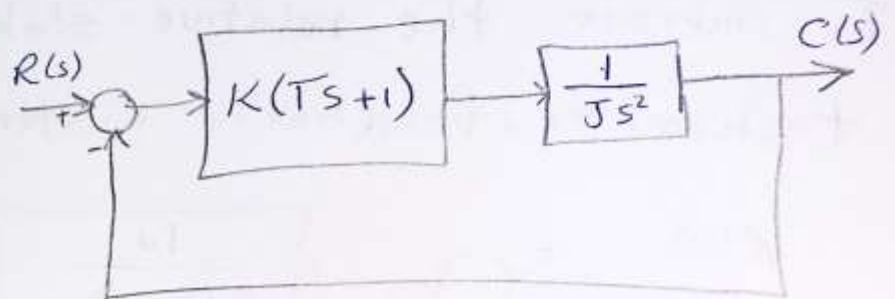
$$\Rightarrow \omega_n = \checkmark$$

[2] sheet 5

3 block diagram of space-vehicle attitude-control system. Assuming the time constant T of the Controller to be 3 sec and the ratio $\frac{K}{J}$ to be $\frac{2}{9} \text{ rad}^2/\text{sec}^2$; Find damping ratio:-

$$T = 3 \text{ sec}$$

$$\frac{K}{J} = \frac{2}{9} \quad \gamma ??$$



$$\frac{\frac{K(Ts+1)}{Js^2}}{1 + \frac{K(Ts+1)}{Js^2}} = \frac{\frac{2}{9} \frac{(3s+1)}{s^2}}{1 + \frac{2}{9} \frac{(3s+1)}{s^2}} \quad \times 9s^2$$

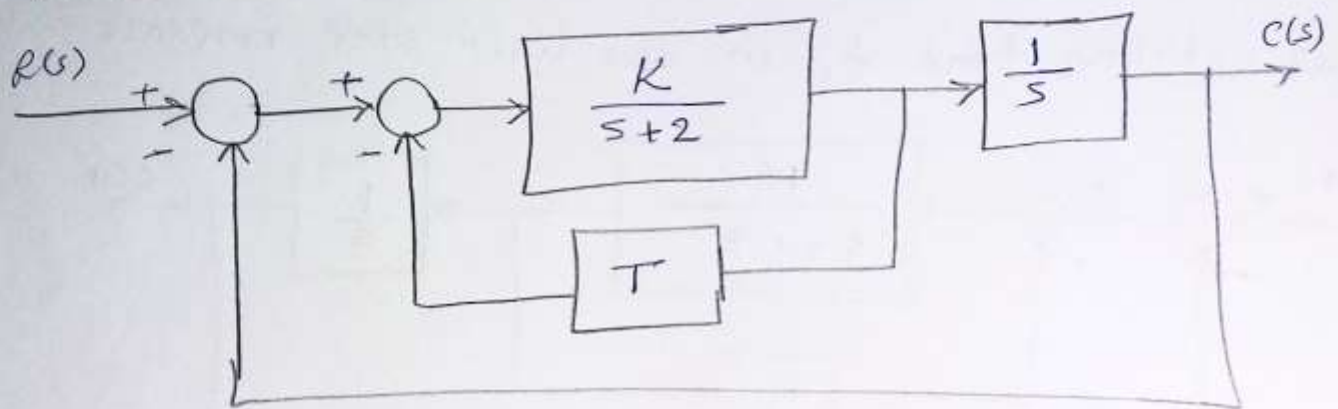
$$\frac{C(s)}{R(s)} = \frac{6s+2}{9s^2+6s+2}$$

$$\omega_n^2 = \frac{2}{9} \longrightarrow \omega_n = \frac{\sqrt{2}}{3} \text{ rad/sec}$$

$$2\gamma\omega_n = \frac{6}{9} \longrightarrow \gamma = \frac{1}{\sqrt{2}}$$

$$\gamma = \frac{1}{\sqrt{2}} < 1 \text{ (damped)}$$

[5] Referring to the system shown in Figure 3, determine the values of K and T such that the system has a damping ratio ζ of 0.7 and an undamped natural frequency ω_n of 4 rad/sec.



$$\Rightarrow \frac{\frac{K}{s+2}}{1 + \frac{KT}{s+2}} \cdot \frac{K}{s+2+KT} \times \frac{1}{s} = \frac{K}{s^2 + KTs + 2s}$$

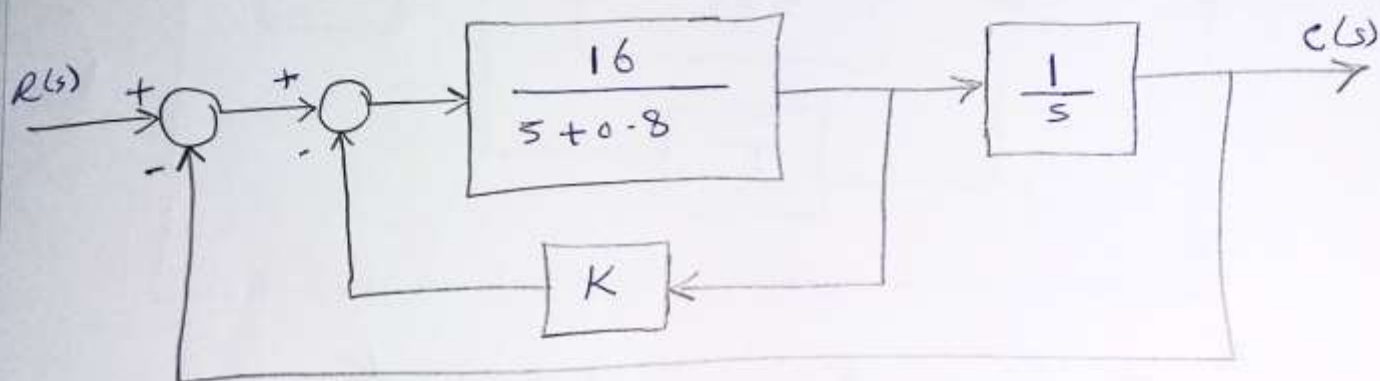
$$\frac{C(s)}{R(s)} = \frac{C(s)}{R(s)} \cdot s \cdot \frac{\frac{K}{s^2 + KTs + 2s}}{1 + \frac{K}{s^2 + KTs + 2s}} = \frac{K}{s^2 + (KT+2)s + K}$$

$$\omega_n^2 = K \Rightarrow \boxed{K = 16}$$

$$KT + 2 = 2\zeta\omega_n = 2 \times 0.7 = 1.4 \times 4$$

$$16T + 2 = 5.6 \Rightarrow \boxed{T = 0.225}$$

6] Consider the system shown in Figure 4. Determine the value of K such that the damping ratio ζ is 0.5. Then obtain the rise time t_r , peak time t_p , maximum overshoot M_p , and settling time t_s in the unit step response.



$$\rightarrow \frac{\frac{16}{s+0.8}}{1 + \frac{16K}{s+0.8}} = \frac{16}{s+0.8+16K} * \frac{1}{s} = \frac{16}{s^2 + 0.8s + 16K}$$

$$\omega_n^2 = 16 \Rightarrow \omega_n = 4 \text{ rad/sec}$$

$$16K + 0.8 = 2\zeta\omega_n = 2 * 0.5 * 4 = 4 \Rightarrow K = 0.2$$

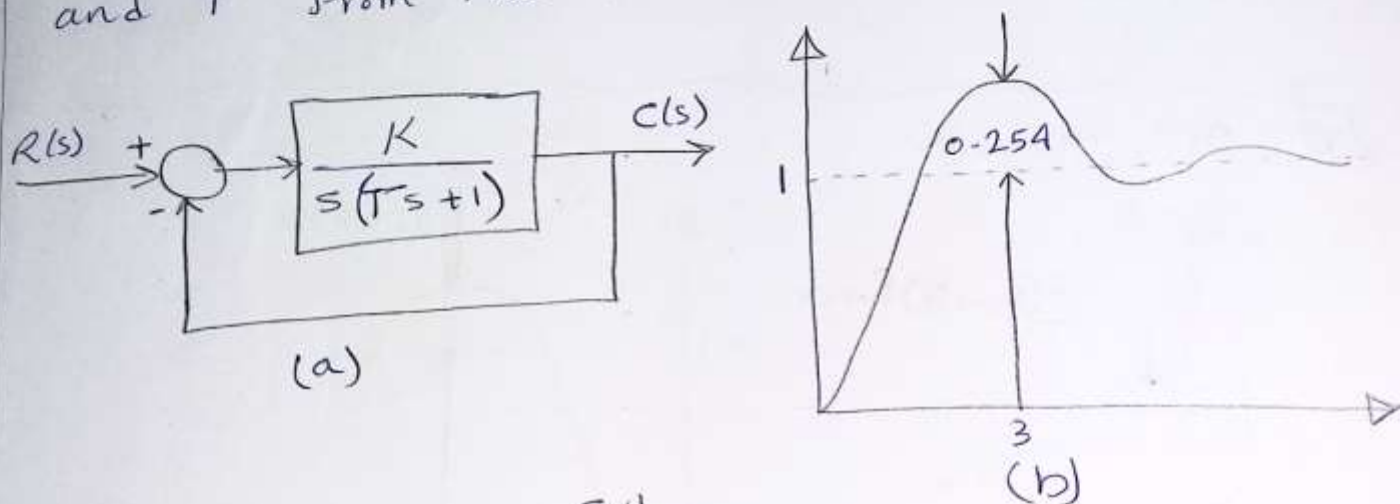
$$t_r = \frac{\pi - \cos^{-1}\zeta}{\omega_n \sqrt{1-\zeta^2}} = \checkmark$$

$$t_s = \frac{4}{\zeta\omega_n} = \checkmark$$

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = \checkmark$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \checkmark$$

7 When the system in Figure (a) is subjected to a unit-step inputs, the system output responds as shown in Figure (b). Determine the values of K and T from the response curve.



Sol

From b ① maximum overshoot $M_p = 0.254$

② Peak time $t_p = 3 \text{ sec}$

$$M_p = e^{-\frac{\gamma}{\omega_n} \pi \sqrt{1-\gamma^2}} = 0.254$$

$$\therefore \gamma = \omega$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\gamma^2}} = \omega \Rightarrow \omega_n = \omega$$

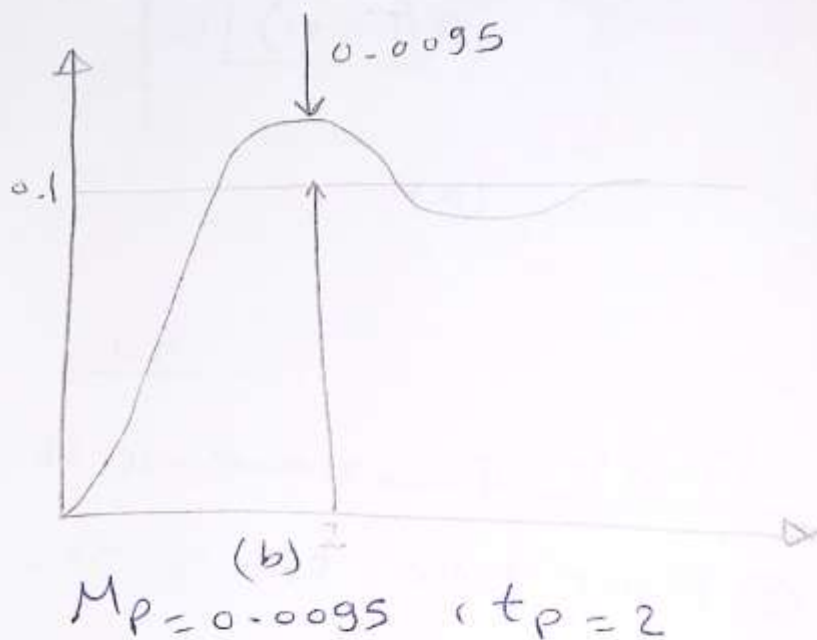
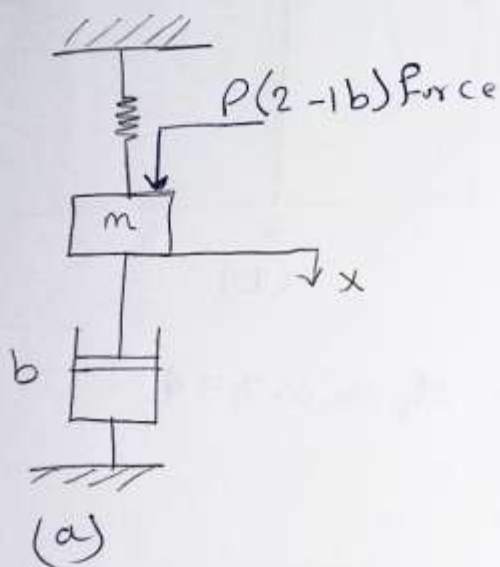
$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(Ts+1)}}{1 + \frac{K}{s(Ts+1)}}$$

$$\frac{C(s)}{R(s)} = \frac{K}{Ts^2 + s + K}$$

$$K = \omega_n^2 \quad (2\zeta\omega_n = 1) \quad \therefore K = \omega_n^2$$

$$T = 1$$

8 get m, b, K



From (b)

$$M_p = e^{-y\pi/\sqrt{1-y^2}} = 0.0095$$

$$\frac{y}{\sqrt{1-y^2}} = 1.48 \Rightarrow y^2 = 2.19 - 2.19y^2$$

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$$y^2(1+2.19) = 2.19 \Rightarrow y = \pm 0.8276$$

$$\boxed{y = 0.827}$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1-\gamma^2}} = 2$$

$$2 = \frac{\pi}{0.56 \omega_n} \Rightarrow \omega_n = 2.79$$

From (a)

$$\frac{X(s)}{P(s)} = \frac{1}{ms^2 + bs + K}$$

$$\omega_n^2 = \frac{K}{m} \Rightarrow \frac{K}{m} = 7.833$$

$$2\zeta\omega_n = \frac{b}{m} = 4.62$$

$$X(s) = G(s) * P(s) \quad , \quad P(s) = \frac{2}{s}$$

$$X(\infty) = \lim_{s \rightarrow 0} s X(s) = \lim_{s \rightarrow 0} \frac{2}{s(ms^2 + bs + K)} = \frac{2}{K}$$

$$\frac{2}{K} = 0.1 \rightarrow \text{Final value from } b$$

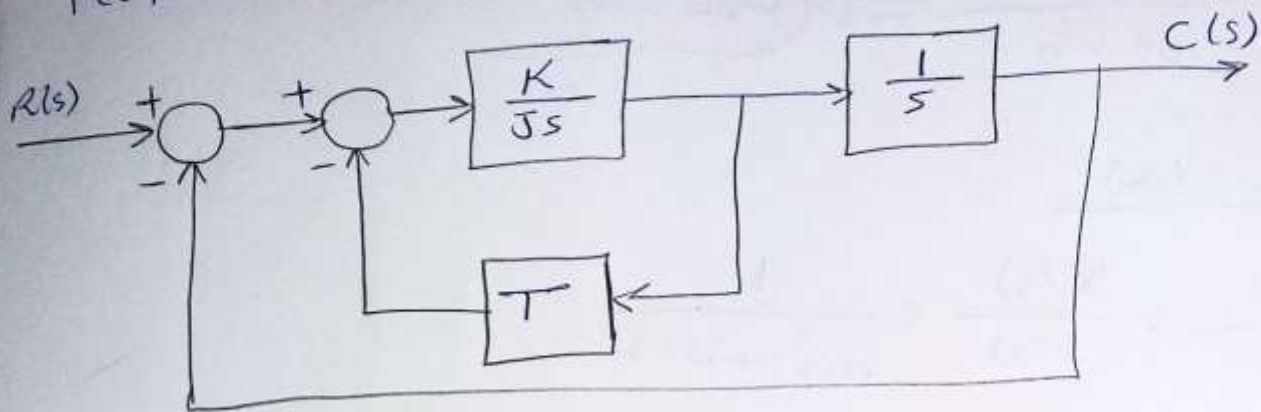
$$K = 20 \Rightarrow \therefore m = 2.55$$

$$\therefore b = 11.796$$

9] Determine values of K, T

maximum overshoot in unit-step response 25%.

Peak time is 2 sec. Assume $J = 1 \text{ Kg-m}^2$



Sol

$$F_1 = \frac{\frac{K}{Js}}{1 + \frac{KT}{Js}} = \frac{K}{Js + KT}$$

$$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(Js + KT)}}{1 + \frac{K}{s(Js + KT)}} = \frac{K}{Js^2 + KTs + K}$$

$$J = 1$$

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + KTs + K}$$

$$M_p = e^{-2\pi/\sqrt{1-y^2}} = \frac{25}{100} = 0.25$$

$$\frac{Z}{\sqrt{1-Z^2}} = 0.44 \Rightarrow Z^2 = 0.19 - 0.19Z^2$$

$$L^2(1 - 0.19) = 0.19 \Rightarrow L = \pm 0.484$$

$$Z = 0.484$$

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$W_n = 1.795$$

$$W_n^2 = K \Rightarrow K = 3.22$$

$$KT = 2\gamma \omega_n$$

$$3.22 \times T = 2 \times 0.484 \times 1.795$$

$$T = 0.5396$$